AE 4d 6
HW #4
Solution

Find Basic Parameters.

$\tau_\lambda = 1 + \frac{\gamma-1}{2} M_0^2 = 1.8$

$\tau_c = (\pi_c)^{\frac{\gamma-1}{\gamma}} = 20^{0.4/1.4} = 2.354$

$\tau_t = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right)(\tau_c - 1) = 0.652$

(a) Find M_9, T_9, u_9

$$M_9 = \left(M_0^2 \frac{\tau_\lambda \tau_c - 1}{\tau_r - 1} \right)^{1/2} = \left(4 \frac{(1.8)(2.354)(0.652) - 1}{(1.8) - 1} \right)^{1/2}$$

$\Rightarrow$ $M_9 = 2.969$ (2)

$$T_9 = T_0 \frac{\tau_\lambda}{\tau_t \tau_c} = (222.2\,K) \frac{1.8}{(1.8)(2.354)} = 367.08$$

$\Rightarrow$ $T_9 = 367.08\,K$ (2)

$$u_9 = M_9 \sqrt{\gamma R T_9} = M_9 \sqrt{(\gamma-1) C_p T_9} = (2.969)\left((0.4)(1004.9 \tfrac{J}{kg\,K})(367.08\,K)\right)^{1/2}$$

$\Rightarrow$ $u_9 = (2.969)(384.12 \tfrac{m}{s}) = 1140.47 \tfrac{m}{s}$

$\Rightarrow$ $u_9 = 1140.47 \tfrac{m}{s}$ (2)

(cont'd 5)

(b) Find $F/\dot{m}$ by 5.9 and 5.20

$$a_0 = \sqrt{(\gamma-1) C_p T} = \sqrt{(0.4)(1004.9)(\tfrac{J}{kg\,K})(222.2K)}$$

$\Rightarrow$ $a_0 = 298.86 \tfrac{m}{s}$

$u_0 = M_0 \times a_0 = 2 \times 298.86 (\tfrac{m}{s}) = 597.71 (\tfrac{m}{s})$.

$\therefore$ (5.9) $\rightarrow$ $\frac{F}{\dot{m}} = u_9 - u_0 = 1140.67 (\tfrac{m}{s}) - 597.71 (\tfrac{m}{s})$

$\Rightarrow$ $\frac{F}{\dot{m}} = 542.76 (\tfrac{m}{s})$. (3)

Eq. 5.20 should give the same (or similar) value. if you go through the calculation.

(c) Find f

$$f = \left(\frac{C_p T_0}{R}\right)(\tau_\lambda - \tau_t \tau_c)$$

$$\Rightarrow f = \frac{(1004.9 \tfrac{J}{kg \cdot K})(222.2\,K)}{(19000 \tfrac{Btu}{lbm})(\tfrac{1055.06\,J}{1\,Btu})(\tfrac{0.454\,lbm}{1\,kg})} (7)(1.8)(2.354),$$

$\Rightarrow$ $f = 0.0139$ (3)

Note: Unit conversion factors are given also in Table 5.2

(d) $TSFC = \frac{f}{F/\dot{m}} = \frac{0.0139}{542.76 (\tfrac{m}{s})} = 2.56 \times 10^{-5} (\tfrac{s}{m}) = 2.56 \times 10^{-5} \tfrac{kg}{N \cdot s}$

$\Rightarrow$ $TSFC = 2.56 \times 10^{-5} \tfrac{kg}{N \cdot s} = 25.6 \tfrac{mg}{N \cdot s}$ (3)