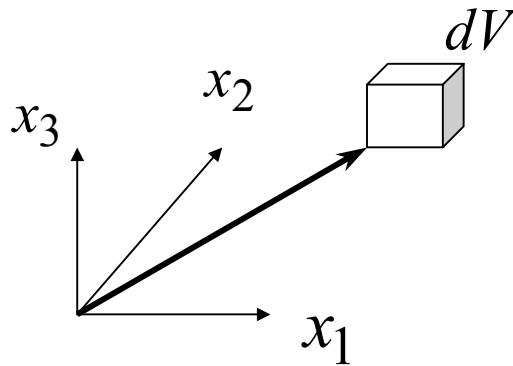


Position and Velocity Distribution Function

In the physical space, for ΔN number of molecules of mass m in a volume of ΔV located at x_i .



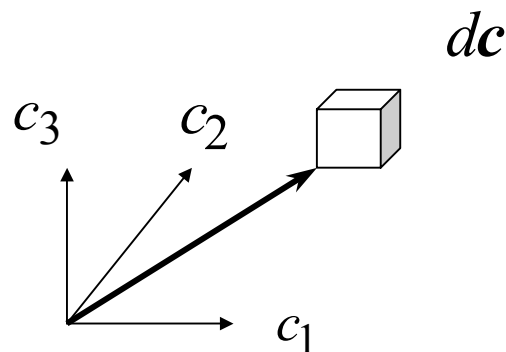
The local mass density is

$$\begin{aligned}\rho(x_i, t) &= \lim_{\Delta V \rightarrow 0} \frac{m \Delta N}{\Delta V} \\ &= m \lim_{\Delta V \rightarrow 0} \frac{\Delta N}{\Delta V} \\ &= m n(x_i, t)\end{aligned}$$

where $dV (\equiv dx_1 dx_2 dx_3)$ and $n(x_i, t)$ represents the local number of molecules found in a unit volume of physical space as a function of space

and time. It is called the local number density or *number density*. Therefore, fluid density is then related to the position distribution function of the number of molecules.

The molecules can also be identified by their three velocity components instead of their three location components in the physical space. In the velocity space, the molecule can be represented uniquely by a point in a Cartesian space with the velocity components as the coordinates. The gas in a region of interest then is represented by a cloud of N points in the velocity space.



Therefore, we can define a velocity distribution function f ,

$$dN = N f d\mathbf{c}$$

where $d\mathbf{c} \equiv dc_1 dc_2 dc_3$ denotes a differential volume in the velocity space. The velocity distribution function provides a statistical description of a gas on a molecular level. We can derive an equation for f using a procedure similar to that used in the derivation of conservation equations at macroscales. The resulting equation, the Boltzmann equation, is

$$\frac{\partial}{\partial t}(nf) + \mathbf{c} \cdot \frac{\partial}{\partial \mathbf{x}}(nf) + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{c}}(nf) = \left\{ \frac{\partial}{\partial t}(nf) \right\}_{collision}$$

The Boltzmann equation is difficult to solve analytically. Numerical simulations such as direct simulation Monte Carlo (DSMC) method and molecular dynamics (MD) method are often used.